Assuming a unit incident voltage wave at the plane of the short circuit in the waveguide, the following algorithm can be used to compute the relevant quantities defined in Fig. 3:

$$V_{inc}^{(0)'} = 1 \tag{5a}$$

$$V_{ref}^{(0)'} = -1 \tag{5b}$$

$$l_0 = 0. \tag{5c}$$

Then,

$$V_{\substack{inc\\ref}}^{(k)} = V_{\substack{inc\\ref}}^{(k-1)'} \exp\left[\frac{\pm j2\pi(l_k - l_{k-1})}{\lambda_g}\right] \tag{6a}$$

$$\substack{e_k\\i_k} = V_{inc}^{(k)} \pm V_{ref}^{(k)} \tag{6b}$$

$$I_1^{(k)} = i_k - jB_{ak}e_k \tag{6c}$$

$$V_1^{(k)} = \frac{I_1^{(k)}}{U_k} \tag{6d}$$

$$e_k' = V_1^{(k)} + e_k \tag{6e}$$

$$i_k' = I_1^{(k)} - jB_{ak}e_k' \tag{6f}$$

$$V_{\substack{inc\\ref}}^{(k)'} = \tfrac{1}{2}[e_k' \pm i_k'] \tag{6g}$$

where $k = 1,2,\cdots,N$, and λ_g is the guide wavelength. The generator voltage E_0, the voltage insertion loss ratio t_k of the kth filter, and the input reflection coefficient ρ are given, respectively, by

$$E_0 = I_1^{(N)} + e_N' \tag{7}$$

$$t_k = 2\left(\frac{1}{R_0}\right)^{1/2}\frac{V_2^{(k)}}{E_0} = -2(R_0)^{1/2}\frac{Y_{12}^{(k)}}{1 + R_0Y_{22}^{(k)}}\frac{V_1^{(k)}}{E_0} \tag{8}$$

$$\rho = \frac{e_N' - I_1^{(N)}}{e_N' + I_1^{(N)}}. \tag{9}$$

Design Procedure

The design procedure, summarized by the flow chart of Fig. 4, is described as follows.

1) Synthesize individual bandpass filters to meet the required selectivity and in-band flatness of the multiplexer specifications [2], [4].

2) Choose initial spacings for the filters. These spacings can be either $l_k = k\lambda_{gk}/2$, or according to the rule, $l_0 = 0$, $l_k - l_{k-1} = \lambda_{gk}/2$, $k = 1,2,\cdots,N$, where λ_{gk} is the guide wavelength at the center frequency f_{0k} of filter number k.

3) Compute the frequency response of the multiplexer using the analysis algorithm described in the previous section.

4) Find j so that

$$|\rho(f_{0_j})| = \max_{k=1,2,\cdots,N} |\rho(f_{0k})|. \tag{10}$$

5) If $|\rho(f_{0_j})| < \epsilon$ (a prespecified allowable reflection coefficient), then all reflection coefficients are acceptable. Print out the results and stop; otherwise, continue to step 6).

6) Change the spacing l_j of filter number j according to the following rule.

If $|\rho(f_{0_j} + \Delta)| \lessgtr |\rho(f_{0_j})|$, then set the new value of l_j equal to $l_j + \Delta l_j$, where

$$\Delta l_j = \lambda_{g_j}\left[1 - \frac{\lambda_g(f_{0_j} \pm \Delta)}{\lambda_{g_j}}\right] \tag{11}$$

and $\lambda_g(f_{0_j} \pm \Delta)$ is the guide wavelength at frequency $(f_{0_j} \pm \Delta)$.

7) If the allowable number of iterations has been exceeded, stop; otherwise, return to step 3).

The convergence of the iteration procedure to an acceptable solution was fairly rapid in all cases tested. This can be attributed to the fact that, although the initial choice of spacings does not produce the desired response, it is not very far from being optimum. The rule for changing the spacings is similar to the procedure for an empirical design approach. Namely, at each step, the filter having the worst return loss is moved by an amount which will move the position of the best return loss to its center frequency.

Example and Discussion

The above procedure has been used in the design of a 6-channel multiplexer. The filters used are 4-pole elliptic-function-type filters having 0.05-dB ripple and 42-MHz bandwidth. The insertion and return losses with the initial spacings and after the application of the optimization procedure are shown in Figs. 5 and 6, respectively. The final result of Fig. 6 was obtained after moving every filter at least twice (two iteration cycles).

The success of the procedure from a practical point of view depends largely on how closely the equivalent circuit models for the filters of Fig. 2 and the junction of [3] represent the actual behavior of these elements over the entire frequency band of the multiplexer. Fig. 7 compares the calculated and measured input reflection coefficients of a typical filter. Although a complete multiplexer assembly designed according to the procedure has not been made, the close agreement of the measured characteristics of a single filter and the computed response indicates that the present approach should yield a satisfactory practical design.

Conclusion

A procedure for the computer-aided design of waveguide multiplexers has been described. This method is based on an analysis algorithm for the equivalent circuit of the multiplexer. Simple rules for the optimization of the filter spacings allow the optimum design to be obtained in a small number of iteration steps. The filters used in the multiplexer can be either direct-coupled (Chebychev) or multiple-coupled (elliptic function) cavity filters.

An example of a 6-channel waveguide multiplexer for the frequency band of 3.7–4.2 GHz is included.

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Microwave Circuit Optimization Employing Exact Algebraic Partial Derivatives

GEORGE R. BRANNER

***Abstract*—A technique for the optimization and sensitivity analysis of broad classes of electrical networks is illustrated. The method utilizes the exact algebraic partial derivatives of functions with respect to any desired independent variable. This completely automated technique has the obvious advantage that the derivatives of any circuit response function with respect to any desired component parameter may be obtained with no additional analytical effort on the part of the designer. Several examples are given to illustrate the procedure.**

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I. Introduction

In recent years there has been growing interest in the application of automated optimization techniques for network and device design [1]. Initially, many problems were solved by employing direct search techniques because the reliable derivative information necessary in gradient methods was difficult to obtain. Bandler [2], [3] has pointed out that there is currently considerable interest in various methods of evaluating the partial derivatives of objective functions and performing sensitivity analysis employing Tellegen's theorem [4]. Recently, considerable effort has been expended toward the development of efficient methods of applying this technique to a broad class of electrical network problems [2]–[4].

The objective of this paper is to present a reliable technique for sensitivity analysis and gradient computation using the exact algebraic partial derivatives of the requisite functions. In the case of the optimization problem, this permits the gradient vector necessary for multidimensional gradient strategies to be evaluated for any microwave circuit response function with respect to any constituent parameter. One such evaluation must be performed for each frequency (or time) increment. The method has been employed with considerable success by the author on both nonlinear [5] and linear active networks and on medical electronics problems [6].

II. The Technique

Exact Partial Derivatives

It is assumed that a Fortran version of the analysis program is in existence. The partial derivatives are obtained from a general computer program specifically written for this purpose. The program accepts as input the complete Fortran subroutine or program describing the problem being investigated preceded by a card which specifies the independent variables of the differentiation. The output of the program is a complete Fortran version of the original program containing all the partial derivatives with respect to the independent variables. This major output is actually a new deck of punched cards which contains a copy of the original deck, all the desired partial derivatives, and a list of numerical codes for all variables [7].

A trivial example illustrating the simplicity of the procedure is shown in Fig. 1. Fig. 1(a) shows a simple Fortran program for which it is desired to obtain all partial derivatives with respect to parameters $X1$ and $X2$. Fig. 1(b) illustrates the differentiation program output. The independent (WRT) variables are listed and numbered in the order in which they appear on the WRT card. Next, the dependent variables are numbered in the order in which they are found in the program. The differentiated program is then listed. Note that the differentials are given names Pxxyy. For example, P0701 is the partial derivative of variable U, which is the objective function in this case, with respect to variable $X1$.

The Optimization Procedure

A weighted least squares optimization scheme is employed to illustrate the efficacy of the technique. The optimization scheme actually employs a modified version of the least squares procedure which handles certain cases where the nature of the multidimensional contour in parameter space may preclude convergence.[1]

Sensitivity Computation

In the examples to be presented in Section III, the classical sensitivity function (1) is employed:

$$S_p{}^R = \frac{p}{R}\frac{\partial R}{\partial p} \tag{1}$$

where

- R circuit response;
- p a circuit parameter;
- $S_p{}^R$ the sensitivity of R to changes in the parameter p.

[1] This is accomplished by employing a procedure, similar to that given in [8], which restricts the maximum per unit change permitted in the parameter values.

```
THETA =  6.28319*X1
THETB =  6.28319*X2
THET1 = SQRT(THETA)
THET2 = SQRT(THETB)
Y1 = COS(THET1)
Y2 = SIN(THET2)
U = (10.*(Y2 -Y1**2))**2  +  (1. -Y1)**2
STOP
END
```

(a)

```
 WRT VARIABLES
                         1                    X1
                         2                    X2

 DEPENDENT VARIABLES
                         1                    THETA
                         2                    THETB
                         3                    THET1
                         4                    THET2
                         5                    Y1
                         6                    Y2
                         7                    U

      THETA =  6.28319*X1
     P0101 = 6.28319
      THETB =  6.28319*X2
     P0202 = 6.28319
      THET1 = SQRT(THETA)
     P0301 = (P0101 / 2.) / SQRT(THETA)
      THET2 = SQRT(THETB)
     P0402 = (P0202 / 2.) / SQRT(THETB)
      Y1 = COS(THET1)
     P0501 = 0. - SIN (THET1) * P0301
      Y2 = SIN(THET2)
     P0602 = COS (THET2) * P0402
      U = (10.*(Y2 -Y1**2))**2  +  (1. -Y1)**2
     P0701 = 200.000* (Y2- Y1**2) * (0. - 2 * Y1* P0501) + 2 * (1.- Y
    11) * (0. - P0501)
     P0702 = 200.000* (Y2- Y1**2) * P0602
      STOP
      END
 DONE
```

(b)

Fig. 1. Simple example illustrating differentiator. (a) Original Fortran program. (b) Differentiator output.

III. Examples

In this section, two examples are presented which illustrate the application of the least squares optimization procedure employing the algebraic derivatives. The first example is a distributed network for which optimization and sensitivity computations are performed. The second example illustrates the application of the technique to provide optimum performance for a capacitive coupled filter.

Example 1

In several recent publications, Bandler has considered the problem of microwave transformer design employing optimization techniques. Although the optimization software employed in this paper does not include the least pth technique, good results have been obtained for this circuit by employing an objective function of the form described by Bandler [9]. The present approach using the algebraic derivatives was applied for the optimization of the 2 section transmission line transformer having a 10:1 load to source impedance ratio over a 100-percent bandwidth [10].

The desired VSWR specification was divided into three equal segments over the normalized frequency range of 0.5–1.5 Hz. Using this response specification, optimum transformers were obtained using the different starting values employed by Bandler [10]. The number of iterations required varied between 51 and 82 depending on the parameter starting values.

The sensitivity functions were calculated from (1) using the algebraic derivatives of the circuit VSWR with respect to characteristic impedance and line length. A plot of these sensitivities for the optimum transformer impedance values versus frequency is shown in Fig. 2.

Example 2

In this example the optimization technique is employed to obtain design values for a lossy 25-element capacitor coupled filter which

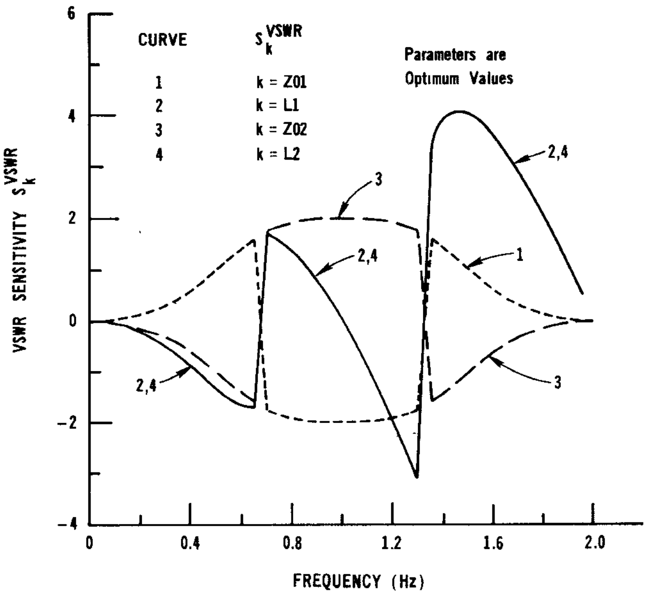

Fig. 2. Sensitivity functions.

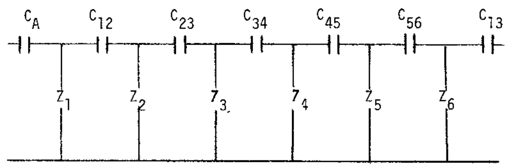

Fig. 3. Capacitive coupled filter. (Each shunt element Z_j represents a parallel connection of L_j, C_j, and R_j.)

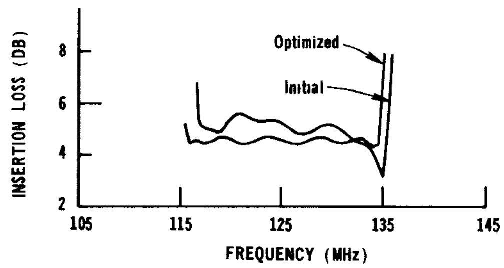

Fig. 4. Insertion loss.

TABLE I
ELEMENT VALUES FOR THE CAPACITIVE COUPLED FILTER

ELEMENT	INITIAL VALUE (pf)	OPTIMIZED VALUE (pf)
C_A	9.857	9.141
C_B	11.277	10.850
C_{12}	4.264	3.960
C_{23}	2.028	1.810
C_{34}	3.703	3.470
C_{45}	2.736	2.730
C_{56}	3.095	2.959
C_1	19.971	20.359
C_2	26.601	26.640
C_3	26.577	27.130
C_4	26.927	26.209
C_5	27.528	26.720
C_6	20.536	20.270

Shunt resonator inductors L_1 through L_6 were 50 nH, and shunt resonator resistances were 5.851 kΩ each These components were constrained during the optimization.

was to operate over a 15-percent frequency band centered around 125 MHz. The filter, which is shown in Fig. 3, was designed to have a 0.1-dB equal-ripple response with a 4.6-dB insertion loss.

Due to filter geometry constraints and the unavailability of a precise design technique, it was necessary to obtain the component values from an approximate narrow-band design approach. When these parameter values were used, a severely distorted filter response as shown in Fig. 4 was obtained.

The optimization technique using the exact algebraic derivatives of the least squares objective function using insertion loss was employed to find component values which would give the desired response. Problem constraints dictated that a subset of 12 of the total 25 elements must be held constant during the optimization procedure. The final response which was obtained after application of the technique is also illustrated in Fig. 4. Initial and final element values are presented in Table I.

IV. Conclusions

This paper details a computer oriented technique for network optimization utilizing exact algebraic partial derivatives of the response function with respect to any circuit parameters of interest. The method is applicable to a broad class of active, nonlinear, and distributed circuits. The method of obtaining the partial derivatives eliminates the disadvantages inherent in the numerical estimation of the derivatives and requires no additional analytical effort. Several examples were presented to illustrate the efficacy of the technique.

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Letters

Group-Delay Smoothing by Noncentral Statistics

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Abstract—**Automatic network analyzer group-delay measurements are improved by simple hardware substitutions, more exact frequency calculations, and a discounting of group-delay variations that fail to persist through several settings of the reference path length.**

Introduction

Severe group-delay flatness requirements are sometimes imposed upon microwave relay components such as band-separation filters. Delay variations of 0.2 ns out of 75 ns must be resolved at frequency intervals of about 1 MHz.

The Hewlett-Packard 8542A general-purpose automatic network analyzer indicates spurious delay variations much larger than the desired tolerances, when used in its stock configuration with two-port measurement programs such as Hewlett-Packard's CGPS2 or Computer Metrics' GPM1. These programs compute group delay from successive CW transmission measurements in accordance with the approximate relation

$$\tau_g = -\frac{\Delta\phi}{360\Delta f} \qquad (1)$$

where $\Delta\phi$ is the phase change in degrees corresponding to a frequency increment Δf. Sources of inaccuracy will be discussed along with techniques for combating them.

Frequency-Set Error

Some of the problems are peculiar to the limited resolution of digital systems. For example, the frequency can be set only to discrete values which are spaced 2–20 KHz apart, depending upon the operating frequency. When the program *calls* for a particular frequency, a phase-locked signal source is *set* to the nearest discrete frequency. A possible discrepancy of 2 KHz is 0.2 percent of a 1-MHz frequency increment, too large to ignore. Therefore, the difference between *set* frequencies must be used as the Δf in (1), whereas the stock software uses the difference between the *called* frequencies.

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Resolver Error

Another digital resolution problem arises in the measurement of phase. The analog-to-digital converter used to read the transmission coefficient in Cartesian form does not have enough bits to give accurate phase differences when the frequency steps are small in comparison to the electrical length of the component under test. In such a situation, some improvement can be expected from averaging results obtained by repeating the measurement with small variations in the size of the frequency steps.

This time-consuming approach was set aside in favor of the following procedure that also bypasses the quadrature imbalance errors inherent in a Cartesian coordinate measurement. The analog phase output of the 8413A phase-gain indicator is read by a precision digital voltmeter interfaced to the system computer. The analog phase detector output is smooth and linear except at the extremes of its range, where it becomes totally nonlinear and changes by an amount corresponding to 360°. One or more such crossovers usually occurs over the passband of a typical communication filter.

Since the results for some of the frequency intervals will be completely obliterated by the crossovers in a single pass over the frequency band, replicate measurements are required with the crossovers somehow shifted to other parts of the band. This shift can be done with the manual phase offset control on the 8413A or by altering the length of the reference signal path with the internally-provided calibrated trombone; the latter procedure was adopted. From the band-center frequency and the number of replications, the computer determines and requests trombone settings that vary the phase over a full cycle.

Information-Processing Rationale

Next, the replicate measurements are dealt with by a simple procedure for using *a priori* knowledge. We know that changing the setting of the trombone does not change the properties of the com-